# Probabilistic analysis in computational mechanics with applications in Civil Engineering

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# Overview

**Research goal: Design techniques for safer and more efficient infrastructure**

**How: Improving probabilistic numerical analysis (specifically for rare event simulation)**

**Contents:**

- Risk and rare event estimation is important in Civil Engineering
- Challenges in probabilistic numerical analysis
- Advances made due to this research

# Consequences of failures in Civil Engineering

**Typical risk profile: low probability, high consequence**

## I-35 Bridge Collapse - 2007

13 fatalities, 100 injured

~$360 million in damages

CNN 2008, url: http://tinyurl.com/jxa9fmw

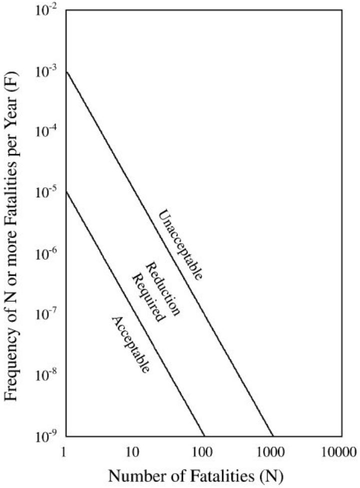
## Teton Dam failure - 1976

11 fatalities

$2 billion in damages

L. Thomson, 1976, url: http://tinyurl.com/jnj6nnq

# Risks in Civil Engineering and acceptable risk




## Working definition of risk

Risk = (Probability of an event) × (consequences of the event).

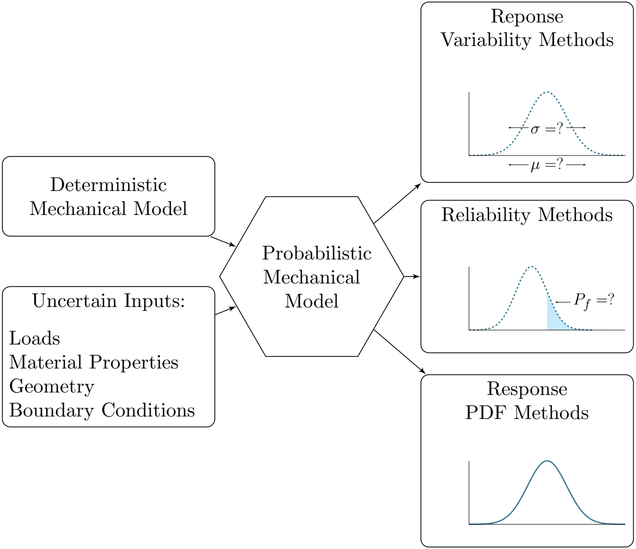
## Risk in Civil Engineering

Unacceptable performance carries severe consequences. Potential for loss of life and large damage cost. This necessitates that the probability of unacceptable performance is small.

## Image: Safety targets for societal risks in The Netherlands

From Faber & Stewart (2003). doi:10.1016/S0951-8320(03)00027-9

# Uncertainty propagation in numerical models

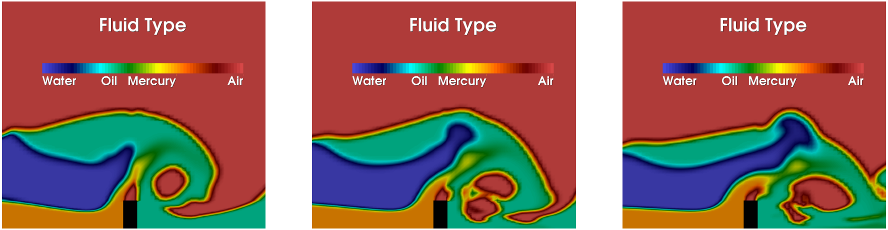


# Deterministic Mechanical Models

**Modern deterministic numerical mechanical models are computationally demanding!**

**Probabilistic analysis needs to handle this complexity.**

Example: Multiphase Finite Volume Model




Moderately computationally demanding for a single analysis

# Probabilistic Mechanical Models

**Probabilistic analysis is even more computationally demanding than deterministic numerical analysis!**

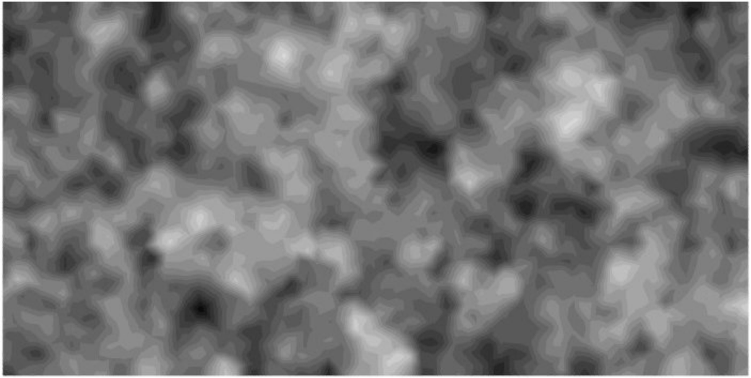
- Have to analyse a range of inputs and then characterise a possible range of outputs to a model
- Simple probabilistic models fail to model input space.
- For many models in Civil Engineering **spatial variability** plays a significant role in constraining the possible output space.

**To accurately pin down output probabilities (and therefore risks) you need to account for spatial variability in material properties**

# Capturing spatial variability with random fields

## Random field simulation - one sample shown below

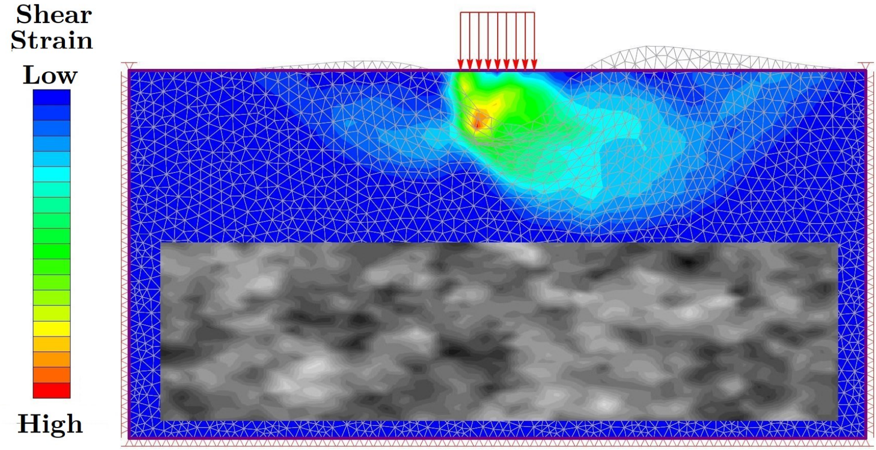
Point variability: Each point has values given by some distribution
Spatial variability: Points nearby are strongly correlated, points at a distance are not correlated.



# Combining numerical models and random fields

## Example: FEM building footing model on soil (nonlinear stress/strain)

Probabilistic models can capture asymmetric response under load



# An early contribution of this work

**An efficient random field simulator**

- Rigorous accuracy/error bounds on the computation.
- Parallel, high performance computing.
- For experts: handles numerically singular covariance matrices while retaining speed of Cholesky decomposition over eigenvalue decomposition for positive definite covariance functions.

**Publication:** Green, Douglas and Mostyn (2015) - *The simulation and discretisation of random fields for probabilistic finite element analysis of soils using meshes of arbitrary triangular elements*, Computers and Geotechnics. **doi:10.1016/j.compgeo.2015.04.004**
**Example application in paper:** Probabilistic finite element analysis for slope stability.

# Existing techniques not well suited to industry applications

**Rare event analysis critical in Civil Engineering**

- Many of the interesting problems in practice involve rare event estimation.
- Rare event modelling - hard problem.

**Existing techniques not well suited to this problem**

- Spectral methods (series expansion methods) - good for mean response. Can only handle simple physical models.
- Monte Carlo Simulation - Good for mean, realistic models, high computational resource demands.
- Subset Simulation - Markov Chain Monte Carlo sampling. Works well in theory but has problems for complex physical models.

# Improving the usability of probabilistic analysis for real problems

**Subset Simulation - Problems**

- Computational complexity of technique is not well understood.
- Make the best choice of Markov Chain Sampler.
- Improve mathematical methods - existing error bound techniques for Subset Simulation are not efficient. Current technique requires problem to be analysed several times!
- Computationally very demanding. Sampling methods require repeated solutions of computationally demanding deterministic problems.

# Recent work on improvements to Subset Simulation

**Some recently presented results in probabilistic analysis:**

- Several improvements to Subset Simulation for nonlinear finite element analysis.
- Presented in April 2016 at *SIAMUQ2016*.
- Conference: *Society and Institute for Applied Mathematics (SIAM) Conference on Uncertainty Quantification* in Lausanne, Switzerland.
- Journal article currently under peer review.

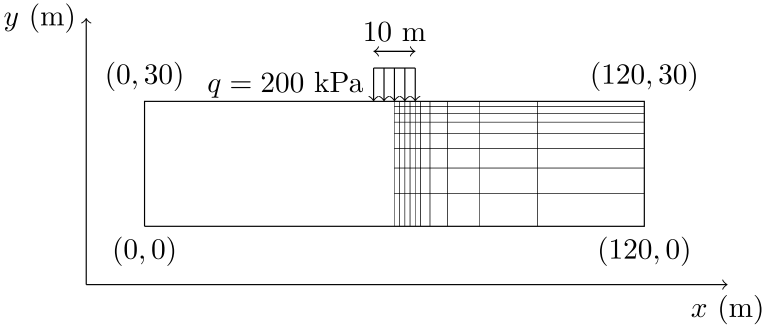
**New results presented at UQ2016:**

- Combined Subset Simulation and random field simulation on a nonlinear finite element problem.
- New methods for calculating the error bounds for Subset Simulation reliability analysis. Can estimate error bounds from single analysis (instead of multiple!).
- Improved understanding of best choice of Markov Chain Monte Carlo sampler for Subset Simulation.

# Subset Simulation - Problem geometry and parameters

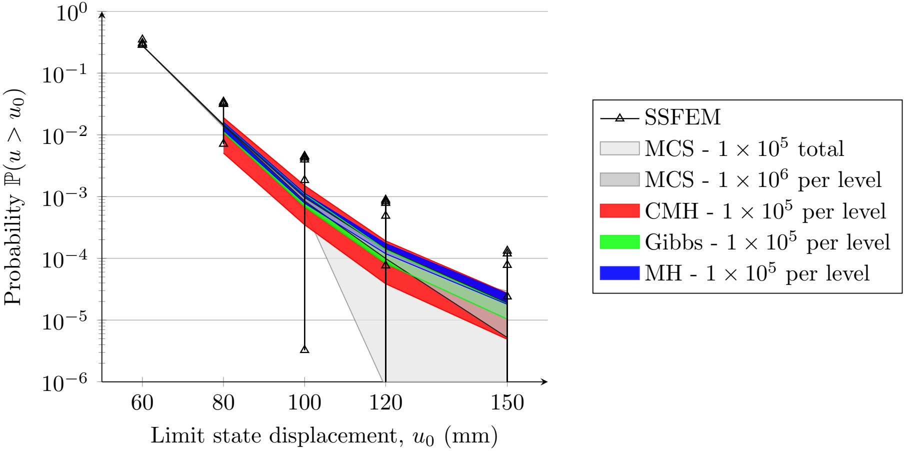
## Existing problem chosen for benchmarking linear case

This problem was then extended to include a nonlinear, random field material model




Mean(E) $= 50$ MPa, CoV(E) $= 0.2$, $\nu = 0.2$.
Originally from: Sudret & Der Kiureghian (2002)
doi:10.1016/s0266-8920(02)00031-0

# Recent contributions - Better MCMC for Subset Simulation




Linear problem. Shaded areas indicate 95% confidence intervals.

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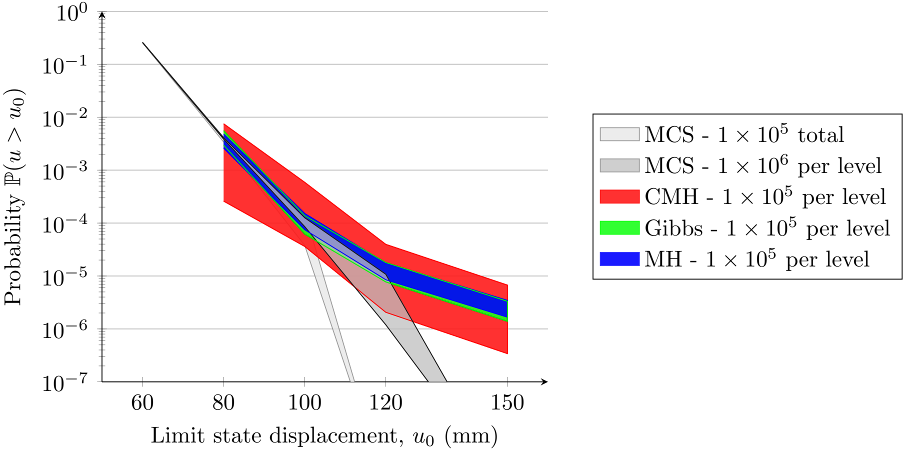


Nonlinear problem. Shaded areas indicate 95% confidence intervals.

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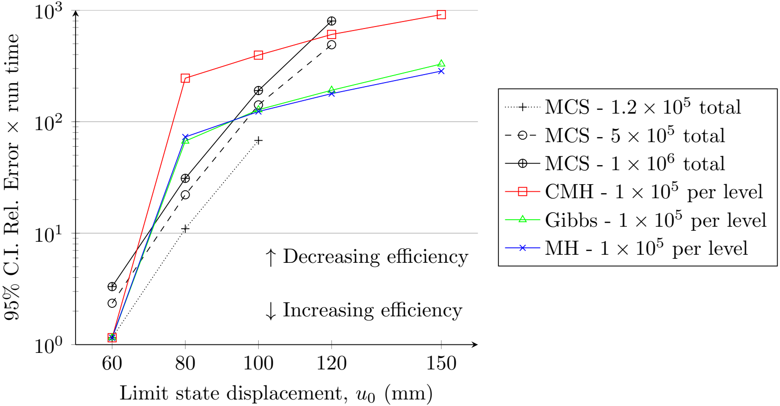



Subset Simulation outperforms Monte Carlo Simulation for rare event simulation (but not for mean response estimation).

Metropolis-Hastings most efficient MCMC tested for nonlinear finite element analysis test problem.

# Conclusions

**Ideal world: zero casualties from infrastructure failures**

**Research contributions**

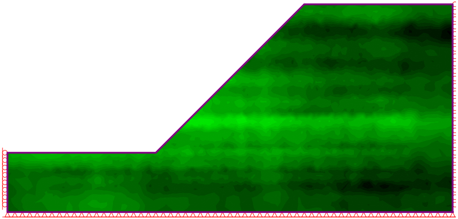
- High performance, modern probabilistic numerical analysis
- Improved techniques for random field simulation
- More efficient rare event probability estimation techniques

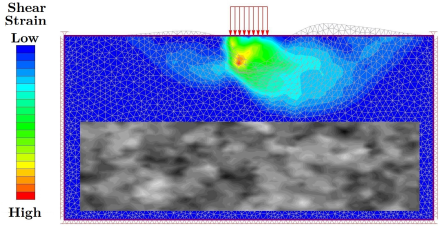
**Impacts**

- Practical and viable rare event simulation.
- Moving towards risk based design in practice.
- Improving design efficiency and safety.

**Improved safety and efficiency for infrastructure are critical for the provision and maintenance of high quality services**

# Thank you!






**Left - random field simulation for slope stability problem from:** Green, Douglas and Mostyn (2015) - *The simulation and discretisation of random fields for probabilistic finite element analysis of soils using meshes of arbitrary triangular elements*, Computers and Geotechnics - **doi:10.1016/j.compgeo.2015.04.004**

**Right - illustration of random material properties under a footing**